CENTRALE NANTES

ENGINEERING PROGRAMME

2024-2025

Year 2 / Year 3

Specialisation option

Mathematics and Applications

OD MATHAPPLI

PROGRAMME SUPERVISOR
Mathieu RIBATET

Autumn Semester

Course unit	ECTS Credits	Track	Course code	Title
UE 73	12	Core course	AF APST1 PROBA	Functional analysis Statistical Learning Probability
		Track: Numerical analysis and probability	ANNUM1	Numerical analysis
		Track: Statistics and Data Science	STAT1	Inferential statistics and linear models
UE 74	13	Core course	MNP P1MATHAPPLI PRSTO	Probabilistic numerical methods Project 1 Stochastic processes
		Track: Numerical analysis and probability	ANEDP ANNUM2	Partial differential equations Advanced numerical analysis
		Track: Statistics and Data Science	SDR STAT2	Data science with R Modelling dependent data

Spring Semester

Course unit	ECTS Credits	Track	Course code	Title
UE 83	14	Core course	APST2 P2MATHAPPLI QI	Advanced statistical learning Project 2 Uncertainty quantification
		Track: Numerical analysis and probability	MBS MODST	Modelling for health and biology Stochastic modeling
		Track: Statistics and Data Science	BAYES FAS	Bayesian statistics Foundations of statistical learning

Functional analysis [AF]

LEAD PROFESSOR(S): Françoise FOUCHER / Joseph VIOLA

Requirements

Objectives

- know the classical integration theorems;
- know standard examples of infinite-dimensional Banach spaces (in particular lp and Lp), handle different topologies on these spaces;
- show that a linear application is continuous, determine its norm;
- know results on orthogonality and orthogonal projection in Hilbert spaces, manipulate Hilbertian bases.

Course contents

1. Reminder of integration
2. Normal vector spaces of finite or infinite dimension
3. Banach spaces, example of lp and Lp spaces
4. Continuity of linear applications, norms of linear operators
5. Hilbert spaces, projection onto a complete convex, Hilbertian bases, Riesz representation, Lax-Milgram

Course material

- François Golse, Analyse réelle et complexe
- Thierry Gallay, Théorie de la mesure et de l'intégration
- Didier Smets, Analyse fonctionnelle
- Cours en ligne de Isabelle Gallagher: https://www.math.ens.psl.eu/~gallagher/ENS-AF1920.php

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	13 hrs	19 hrs	0 hrs	0 hrs	0 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Autumn Semester - Course Unit 73 / 93

Statistical Learning [APST1]

LEAD PROFESSOR(S): Claire BRECHETEAU

Requirements

Objectives

This lecture is an introduction to statistical learning.
Main objectives:
- understanding the main concepts of statistical learning
- introduction to standard methods in statistical learning
- practice on real data using standard Python libraries

Course contents

- introduction to statistical learning
- standard methods for classification
- Estimating model errors and tuning methods
- CART and random forests
- Introduction to Deep Learning (Feed Forward and CNNs)
- Classical and advanced methods of non supervised learning

Course material

- The Elements of Statistical Learning, Data Mining, Inference, and Prediction. Trevor Hastie, Robert Tibshirani, Jerome Friedman, 2009 Springer.
- Hands-On Machine Learning with Scikit-Learn and TensorFlow. Aurélien Géron, O'Reilly 2017.

Assessment

Collective assessment: EVC 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	0 hrs	18 hrs	0 hrs	0 hrs

Probability [PROBA]

LEAD PROFESSOR(S): Mathieu RIBATET / Nicolas PETRELIS

Requirements

Objectives

Understand the foundations of probability theory.
Learn techniques that are necessary for statistics.
Understand the different notions of convergence of a random variable.

Course contents

- Introduction to measure theory, classical theorems of integration with respect to a general measure.
- Real-valued random variables, usual probability laws, cumulative distribution function, expectation, variance, L^p spaces.
- Vector-valued random variables, independence, characteristic function.
- Convergence of random variables, almost sure, in probability, L^p, and in law. Illustration with the law of large numbers, the central limit theorem.

Course material

Barbe-Ledoux, Probabilités (EDP-sciences)

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	16 hrs	0 hrs	0 hrs	2 hrs

Inferential statistics and linear models [STAT1]

LEAD PROFESSOR(S): Bertrand MICHEL

Requirements

Objectives

Introduce basic statistical concepts and establish asymptotic properties of the proposed methods.

Course contents

- Statistical model and inference
- Statistical inference: moment methods, maximum likelihood, delta-method, asymptotic properties
- Confidence regions
- Linear model

Course material

B. Cadre, C. Vial. 2012. Statistique mathématique - Master 1 et Agrégation, Ellipse.

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	16 hrs	14 hrs	0 hrs	0 hrs	2 hrs

Numerical analysis [ANNUM1]

LEAD PROFESSOR(S): Françoise FOUCHER

Requirements

Objectives

- Know the different classes of equations (linear PDEs) and the model equations (stationary and non-stationary heat, transport, waves)
- Know how to construct a finite difference scheme, and study its properties of consistency, stability and convergence
- Know how to implement a numerical solution on a computer
- Know how to analyze results, quantify errors

Course contents

- PDEs and mathematical models
- Stationary heat equation, finite difference method, consistency, stability and convergence
- Heat equation (non-stationary), explicit and implicit finite difference schemes
- Transport equation, characteristic curves, finite difference schemes
- Wave equation

Course material

- Grégoire Allaire. "Analyse numérique et optimisation". Ellipses, 2005.
- Ionut DANAILA, Pascal JOLY, Sidi Mahmoud KABER, Marie POSTEL. "Introduction au calcul scientifique par la pratique". Dunod, Sciences Sup, 2005.
- Daniel EUVRARD. "Résolution numérique des équations aux dérivées partielles". Masson, 3rd edition, 1994.
- Mark H. HOLMES. "Introduction to numerical methods in differential equations". Springer, 2007.
- Randall J. LEVEQUE. "Finite difference methods for ordinary and partial differential equations". SIAM, 2007.
- Brigitte LUCQUIN. "Equations aux dérivées partielles et leurs approximations". Ellipses, 2004.
- Bijan MOHAMMADI, Jacques-Hervé SAIAC. "Pratique de la simulation numérique". Dunod, 2003.
- Lionel SAINSAULIEU. "Calcul scientifique". Dunod, Sciences Sup, 2000.
- Eleuterio F. TORO. "Riemann solvers and numerical methods for fluid dynamics". Springer, 3rd edition, 2010.

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	16 hrs	0 hrs	0 hrs	2 hrs

Probabilistic numerical methods [MNP]

LEAD PROFESSOR(S): Anthony NOUY

Requirements

Fundamentals in probability, linear algebra and numerical linear algebra (bachelor level).

Objectives

At the end of the course, the student will be able to understand and use stochastic methods for estimating quantities that are expressed as mathematical expectations. He/She will be able to propose and implement a simulation method for generating a sample from a certain distribution or a Markov chain allowing to infer the target quantity, and to evaluate the the precision of the estimation. Finally, he/she will be able to understand and use randomized numerical methods for solving high dimensional problems in scientific computing and data science which can not be treated with classical method in numerical linear algebra.

Course contents

The first part of this course deals with the main methods of simulation of random variables: generation of pseudo-random numbers, inverse cdf method, rejection method and simulation of Markov chain with finite state space. The course then presents the Monte Carlo and MCMC methods as well as variance reduction techniques.
The last part of the course deals with randomized numerical linear algebra methods for high dimensional problems. It will present the principles of parsimonious sampling and random projection methods, and their applications to performing algebraic operations, matrix factorization, the solution of least squares problems and data compression.

Course material

- J. E. Gentle. Random number generation and Monte Carlo methods. Springer Science & Business Media, 2006.
- Rubinstein, R. Y., & Kroese, D. P. (2016). Simulation and the Monte Carlo method (Vol. 10). John Wiley & Sons.
- S. Boucheron, G. Lugosi, and P. Massart. Concentration inequalities: A nonasymptotic theory of independence. Oxford university press, 2013.
- R. Vershynin. High-dimensional probability: An introduction with applications in data science, volume 47. Cambridge University Press, 2018.
- Martinsson, P., & Tropp, J. (2020). Randomized numerical linear algebra: Foundations and algorithms. Acta Numerica, 29, 403-572.

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	0 hrs	16 hrs	0 hrs	2 hrs

Project 1 [P1MATHAPPLI]

LEAD PROFESSOR(S): Mathieu RIBATET

Requirements

Objectives

This course aims to apply the academic lectures of the specialisation in Mathematics and Applications to practical cases or to deepen some fundamental concepts.

The projects will be chosen by the students from a list of research or industrial projects which are related to the research activities of the teaching team.

Course contents

Course material

Assessment

Collective assessment: EVC 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	1	0 hrs	0 hrs	0 hrs	32 hrs	0 hrs

Stochastic processes [PRSTO]

LEAD PROFESSOR(S): Mathieu RIBATET / Philippe CARMONA

Requirements

Objectives

The aim of this course is to describe the stochastic processes that are commonly used in the mathematical modeling of random phenomena which evolve over time (or space). We will study basic processes, show how they are involved in the modeling of systems, and finally solve some problems, mostly related to limit theorems.

Acquired skills: Establish a simple (or simplified) mathematical model of a complex random system. Illustrate the properties of this model by showing properties in finite horizon or asymptotic properties, and interpret these properties by simulations of random processes.

Course contents

1. Poisson and Renewal Processes
2. Markov Chains
3. Martingales

Practical sessions: simulation of complex processes, illustration of limit theorems.

Course material

1. R. Durrett &guil;Essentials of stochastic Processes&guil;, Series: Springer Texts in Statistics, 2nd ed., 2012 ISBN-10: 1461436141
2. H. TIJMS. &guil;A First Course in Stochastic Models&guil;, Wiley, 2nd edition, 2003. ISBN-10: 047149880
3. E. Pardoux. &guil;Processus de markov et applications&guil;, Collection: Sciences Sup, Dunod 2007. EAN13 : 9782100512171

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	10 hrs	6 hrs	0 hrs	2 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Autumn Semester - Course Unit 74 / 94

Data science with R [SDR]

LEAD PROFESSOR(S): Aymeric STAMM / Mathieu RIBATET

Requirements

Objectives

This course is intended for beginners in R and aims to provide the tools for data analysis with R using the most recently developed library suites from the R user community. The course is divided into two parts.

In the first part, an introduction to R is provided over the first 15 hours, to learn the language, data management and exploratory data analysis. In this first part, the emphasis is on practice with 10 hours of practical work for only 5 hours of lectures.

The second part focuses on how to do modelling with R. In other words, once we have established that we want to analyse our data with a certain approach (be it hypothesis testing, linear regression, principal component analysis or clustering), how do we use R to avoid manual calculations? With this in mind, we will (re)examine the assumptions behind each of these statistical methods (10 hours of lectures). We will use R to (i) help us in the computational parts and (ii) help us to validate the hypotheses of our models (5 hours of practical classes).

Course contents

Data Management and Exploratory Data Analysis with R

- Introduction to the tidyverse, a set of packages that work together as an ecosystem with the same philosophy, grammar and data structures to give the best possible user experience for data manipulation and exploratory analysis tasks.
- Visualisation with the ggplot2 package.
- Data manipulation with the dplyr package: variable or observation selection, construction of statistical summaries, addition of derived variables, merging of several datasets, sorting by values of certain variables.
- Transformation of data sets with the tidyr library.
- Management of different types of variables: numeric (R base), strings (stringr), factors (forcats), date/time (hms, lubridate).
- Optimal use of lists and list-variables with the purrr package.
- Automated and interactive reports with text, codes and equations and interactive presentations with Quarto.

Modeling with R

- Hypothesis testing
- Linear regression
- Consistent extraction of relevant information from given models with the modelr package.
- Principal component analysis.
- Partition-based clustering and hierarchical clustering.
- Introduction to tidymodels, a set of packages for modelling and machine learning. These packages also share the same philosophy, the same grammar and the same data structures, in line with the tidyverse.

Course material

https://www.tidyverse.org
https://ggplot2-book.org
https://r4ds.had.co.nz
https://www.rstudio.com/resources/cheatsheets/

https://www.tidymodels.org
https://www.tmwr.org
https://quarto.org

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	15 hrs	0 hrs	15 hrs	0 hrs	2 hrs

Modelling dependent data [STAT2]

LEAD PROFESSOR(S): Mathieu RIBATET

Requirements

Objectives

Get the fundamental framework on 2 statistical themes : times series and geostatistics. Be able to apply the above theory on various datasets.

Course contents

This course has 2 independent chapters:

- Time series: time series definition, weak and strong stationnarity, how to stationnarize a sérieusement, (partial) auto-correlation functions, AR and MA models, ARIMA and SARIMA

- Geostatistics : spatial process, stationarity and isotropy, covariance function and its properties, semi-variogram, kriging, simulation

Course material

- Time series:

P.J. Brockwell and R.A. Davis. Time Series: Theory and Methods. Springer Series in Statistics. Springer, 2009.

P.J. Brockwell and R.A. Davis. Introduction to Time Series and Forecasting. Springer Texts in Statistics. Springer International Publishing, 2016.

Robert Shumway and David Stoffer. Time Series Analysis and Its Applications With R Examples, volume 9. 01 2011.

- Geostatistics :

Chiles, J.-P. and Delfiner, P. (1999). Geostatistics: Modelling Spatial Uncertainty. Wiley, New York.

Cressie, N. A. C. (1993). Statistics for Spatial Data. Wiley Series in Probability and Statistics. John Wiley & Sons inc., New York.

Diggle, P., Ribeiro, P., and Justiniano, P. (2007). Model-based Geostatistics. Springer Series in Statistics. Springer.

Stein, M. L. (1999). Interpolation for spatial data: Some theory for kriging. Springer, New York.

Wackernagel, H. (2003). Multivariate geostatistics: An introduction with applications. Springer, New York, third edition edition.

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	16 hrs	14 hrs	0 hrs	0 hrs	2 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Autumn Semester - Course Unit 74 / 94

Partial differential equations [ANEDP]

LEAD PROFESSOR(S): Françoise FOUCHER / Joseph VIOLA

Requirements

Objectives

The aim of this course is to provide the main tools of mathematical PDE analysis arising from physics and mechanics models. For that, we provide fundamental classical theorems for a rigorous justification of variational approaches.

Course contents

1. Introduction: weak derivations, smooth functions
2. Sobolev spaces: injection, trace
3. Variational formulation for second-order elliptic equations: boundary condition (Dirichlet, Neumann, Fourier, etc)
4. Nonlinear evolution PDE: exact solutions.
5. Linear evolution equations: energy estimates, Galerkin method, maximum principle
6. Project

Course material

- JM GILSINGER, M.JAI. Eléments d'analyse fonctionnelle, Fondements et application de l'ingénieur. Presses Polytechniques et universitaires romandes
- H. BREZIS. Analyse fonctionnelle, Théorie et applications. Masson.
- B. Lucquin. EDP et leurs approximations. Mathématiqes à l'université, ellipses.
- L. C. Evans. Partial Differential Equations, Graduate Studies in Mathematics, Volume 19, AMS.
--Raviart, Thomas, Introduction à l'analyse numérique des   équations aux dérivées partielles, 1998

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	18 hrs	12 hrs	0 hrs	0 hrs	2 hrs

Advanced numerical analysis [ANNUM2]

LEAD PROFESSOR(S): Mazen SAAD

Requirements

Objectives

This course presents mathematical aspects of variationnal approximation method forpartial differential equations. As applications, we will focus on construction, the convergence analysis and the programming of the finite element method (FEM) .

Course contents

1. Complements in functionnal analysis (Lax-Milgram)
2. Galerkin method (Céa, Strang, ..)
3. Finite element (Interpolation, finite element, mesh, error)
4. Finite elements method for second order problems 1D, 2D, 3D

Lab with Matlab and Python

Course material

[1] Raviart, T., Thomas, J.-M., Introduction a l'analyse numerique des equations aux derivees partielles, Dunod (2004)
[2] Brezis, H.,Analyse Fonctionnelle, Dunod (2005)
[3] Ern, A., Guermond, J.-L., Theory and practice of finite elements, Springer (2004)
[4] Haasdonk, B., Reduced Basis Methods for Parametrized PDEs – A Tutorial Introduction for Stationary and Instationary Problems (2014)

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	12 hrs	4 hrs	0 hrs	2 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Spring Semester - Course Unit 103 / 83

Advanced statistical learning [APST2]

LEAD PROFESSOR(S): Mathieu RIBATET

Requirements

Objectives

Study and practice of standard and more advanced algorithms in statistical learning

Practice of standard algorithms with Scikit-learn and Keras.

Course contents

SVM and kernel methods

Boosting methods

Deep Learning: RNNs, NLP and Autoencoders

Reinforcement Learning

Course material

- The Elements of Statistical Learning, Data Mining, Inference, and Prediction. Trevor Hastie, Robert Tibshirani, Jerome Friedman, 2009 Springer.
- Hands-On Machine Learning with Scikit-Learn and TensorFlow. Aurélien Géron, O'Reilly 2017.
- Deep Learning. Ian Goodfellow , Yoshua Bengio, Aaron Courville, 2016.

Assessment

Collective assessment: EVC 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	16 hrs	0 hrs	16 hrs	0 hrs	0 hrs

Project 2 [P2MATHAPPLI]

LEAD PROFESSOR(S): Mathieu RIBATET

Requirements

Objectives

This course aims to apply the academic lectures of the specialisation in Mathematics and Applications to practical cases or to deepen some fundamental concepts.

The projects will be chosen by the students among a list of research or industrial projects which are related to the research activities of the teaching team.

Course contents

Course material

Assessment

Collective assessment: EVC 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	2	0 hrs	0 hrs	0 hrs	48 hrs	0 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Spring Semester - Course Unit 103 / 83

Uncertainty quantification [QI]

LEAD PROFESSOR(S): Anthony NOUY

Requirements

Fundamentals in numerical analysis, probability and statistics (master level)

Objectives

The aim of the course is to introduce fundamental concepts and mathematical tools for the modeling and quantification of uncertainty in predictive science and engineering, and to provide students with a theoretical understanding of numerical methods for stochastic and parametric models.

Acquired skills: tools for probabilistic modeling of uncertainties, uncertainty propagation methods, rare events estimation methods, robust optimization methods, tools for sensitivity analysis, approximation and model order reduction methods

Course contents

- Introduction to uncertainty quantification
- Uncertainty modelling
- Monte-Carlo methods, application to rare event estimation
- Sensitivity analysis and specific methods
- Robust optimization methods
- Approximation of models: elements of approximation theory, interpolation, learning, model order reduction

Course material

- Ghanem, R., Higdon, D., & Owhadi, H. (Eds.). (2017). Handbook of uncertainty quantification (Vol. 6). New York: Springer.
- Rubinstein, R. Y., & Kroese, D. P. (2016). Simulation and the Monte Carlo method (Vol. 10). John Wiley & Sons.

Assessment

Collective assessment: EVC 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	22 hrs	0 hrs	10 hrs	0 hrs	0 hrs

Bayesian statistics [BAYES]

LEAD PROFESSOR(S): Mathieu RIBATET

Requirements

Objectives

This lecture is about Bayesian statistics and its modern implementation. The main goal is to familiarize students to Monte Carlo Markov Chain samplers and be able to implement them on complex statistical models (including hierarchical models). The student will be able to conduct a whole Bayesian analysis starting from mathematical derivations, software implementation and finally doing inference.

Course contents

- Foundation of Bayesian statistics
- Asymptotic behavior of the posterior
- Aim of MCMC samplers
- The need for thinning and burnin
- Usual proposal kernel
- Concrete implementation
- Gibbs sampler
- Concrete implementation
- Hierarchical model and DAG
- Applications on complex models
- If possible, mixture models

Course material

M. K. Cowles. Applied Bayesian Statistics with R and OpenBugs Examples. Springer Texts in Statistics. Springer-Verlag, 2013.

J. A. Hartigan. Bayes Theory. Springer Series in Statistics. Springer-Verlag, 1983.

C.P. Robert. The Bayesian Choice: A Decision-theoretic Motivation. Springer Texts in Statistics. Springer-Verlag, 2007.

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	14 hrs	16 hrs	0 hrs	0 hrs	2 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Spring Semester - Course Unit 103 / 83

Foundations of statistical learning [FAS]

LEAD PROFESSOR(S): Bertrand MICHEL

Requirements

Objectives

This course presents some mathematical approaches to statistical learning and machine learning.

The main objective of the course is to build statistical guarantees on prediction algorithms (risk bounds).

Course contents

- Statistical learning concepts
- Empirical risk minimization
- Vapnick-Chervonenkis dimension
- RKHS
- Introduction to high-dimensional statistics: analysis of Lasso type algorithms.

Course material

Introduction to High-Dimensional Statistics. Christophe Giraud, 2015. Chapman and Hall/CRC.

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	16 hrs	14 hrs	0 hrs	0 hrs	2 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Spring Semester - Course Unit 103 / 83

Modelling for health and biology [MBS]

LEAD PROFESSOR(S): Mazen SAAD

Requirements

Objectives

The aim of this course is the numerical study of the finite volume method applied to some models arising from population dynamics. The models derive from the epidemiology spread disease like influenza, the Norovirus and the FIV (Feline Immunodeficience Virus), the interaction between species as a predator-prey system, chemotaxis as Keller-Segel model, bone regeneration, or breast cancer.

Chemotaxis is the movement of biological individuals towards a chemoattractant ((or away from chemorepellent). A vital characteristic of living organisms is the ability to sense signals in the environment and adapt their movement accordingly. A typical model describing chemotaxis is the Keller-Segel system. This model leads to a nonlinear parabolic system which is the cornerstone of this course.
In this course, we first start with an introduction to models in population dynamics. Next, we introduce the finite volume method on orthogonal mesh and we give the basic discrete functional analysis. Then, a combined finite element/finite volume scheme is introduced to approximate the solutions of standard convective diffusive equations. Finally, the scheme is generalized to the Keller-Segel system and the convergence analysis is presented.

Course contents

1. Models in population dynamics: epidemic models (SIR, influenza...), Keller-Segel models in chemotaxis, bone regeneration, dynamic of solid tumor
2. An introduction to the Finite volume scheme for the diffusion equation
3. Finite volume scheme for the transport equation
4. Combined finite element/finite volume scheme for convection-diffusion equations
5. Combined finite element/finite volume scheme for the nonlinear Keller-Segel model

Course material

[1] Eymard, Gallouet, Herbin: Finite volume method, hal-02100732
[2] Murray J.D.: Mathematical Biology I: An Introduction, Interdisciplinary Applied Math; Springer
[3] Murray J.D.: Mathematical Biology II: Spatial Models and Biomedical Applications, Interdisciplinary Applied Mathematics; Springer
[4] A.M. Turing: The chemical basis of Morphogenesis.1952
[4] L.C. Evans: Partial Differential Equations. Graduate Studies in Math., Vol 19, AMS

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	18 hrs	8 hrs	4 hrs	0 hrs	2 hrs

ENGINEERING - OD MATHAPPLI

Year 2 / Year 3 - Spring Semester - Course Unit 103 / 83

Stochastic modeling [MODST]

LEAD PROFESSOR(S): Antonio FALCO MONTESINOS / Marie BILLAUD

Requirements

Objectives

The goal of this course is to introduce the necessary tools to describe continuous time stochastic models and their relationship with partial differential equations. Also, we discuss numerical method to simulate such processes. In particular, we revisit numerical solutions for PDEs with probabilist point of views. These tools will be illustrated on models from physics, economics and biology for example.

Course contents

1. Brownian motion : stochastic process, definitions, simulation
2. Ito stochastic integral and process : Théorème de Girsanov
3. Stochastic differential equations : generalities, strong and weak solution, diffusion process, numerical integration (Euler Maruyama, Milstein), convergence
4. Probabilistic PDEs representation: heat equation and brownian motion, Feynman Kac representation theorem

Course material

[1] F. Comets, T. Meyre, Calcul stochastique et modèles de diffusion, Dunod, 2015
[2] E. Gobet, Méthodes de Monte-Carlo et processus stochastiques : du linéaire au non linéaire, Editions de l'école polytechnique, 2013
[3] P.E. Kloeden, E. Platen, Numerical Solution of Stochastic Differential Equations, Springer Verlag, 1999
[4] G. Lord, C. Powell, T. Shardlow, Introduction to computational stochastic PDEs, 2010
[5] D. Nualart and E. Nualart, Introduction to Malliavin Calculus: Institute of Mathematical Statistics. Textbooks, Series Number 9, Cambridge University Press, 2018
[6] G. D Pratto, Introduction to Stochastic Analysis and Malliavin Calculus, 2014

Assessment

Individual assessment: EVI 1 (coefficient 1.0)

LANGUAGE OF INSTRUCTION	ECTS CREDITS	LECTURES	TUTORIALS	LAB	PROJECT	EXAM
French	3	18 hrs	12 hrs	0 hrs	0 hrs	2 hrs