SHAKE THE FUTURE.

CENTRALE NANTES

ENGINEERING PROGRAMME

SPECIALISATION

MATHEMATICS AND APPLICATIONS

AUTUMN SEMESTER

Établissement public sous tutelle du
ministère de l'Enseignement supérieur, de la Recherche et de l'Innovation

GROUPE **CENTRALE**

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HILBERTIAN ANALYSIS

CORE COURSE

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Benoit GREBERT

Objectives

At the conclusion of this course, the student will:

- know standard examples of infinite dimentional Banach spaces (such as lp and Lp),
- handle different topologies on these spaces;
- determine if a linear application is continuous;
- handle geometry concepts coming from Hilbert spaces, such as projection;
- handle Fourier series and transforms;
- make the distinction between strong convergence and weak convergence in Hilbert spaces;
- handle easy distributions;
- calculate limits in the sense of distributions, derivatives in the sense of distributions;
- give examples of functions in Sobolev spaces;
- calculate the weak derivative.

Course contents

1. Normed vectors spaces in any dimension. Banach spaces. Example of lp and Lp spaces. Continuity of linear applications between normed vector spaces. Fixed point theorem.
2. Hilbert spaces, projection on a convex and complete space, orthonormal bases in Hilbert spaces. Gram-Schmidt, Riesz representation, Lax-Milgram.
3. Fourier series and transforms.
4. Weak convergence in Hilbert spaces.
5. Introduction to distributions and Sobolev spaces.

Course material

- Cours d'Analyse, Jean-Michel Bony, Editions de l'Ecole Polytechnique, 2001.
- E. Lieb and M. Loss, Analysis, AMS graduate studies in maths (2001)
- Claude Zuily, Eléments de distributions et d'équations aux dérivées partielles. Dunod (2002)
- Isabelle Gallaghers online course: http://www.math.jussieu.fr/~gallagher/chap3.pdf

Keywords

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	14,7 hrs	0,0 hrs	4 hrs

DETERMINISTIC NUMERICAL METHODS

CORE COURSE

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Helene MATTHIS

Objectives

At the end of this teaching module, the student should be able to, with regard to approximation techniques:

- Build and implement the Lagrange interpolation of a function, its polynomial of best approximation and its spline interpolant,
- Identify the relevance of an approximation method compared to another and give qualitative and quantitative interpretation of the numerical results

With regard to numerical linear algebra, the student should be able to:

- Implement the QR factorization to solve overdetermined linear systems for instance for a least square approximation
- Reduce a matrix under a diagonal profile by means of a singular value decomposition.

Course contents

Approximation:

- General notions: polynomial basis of approximation, error, best approximation
- polynomial approximation, trigonometric poynomials (FFT), introduction to wavelets

Interpolation:

- Lagrange, piecewise Lagrange, Hermite, splines
- Least square method, regularized least square method (radial basis, link with interpolation…)

Advanced numerical linear algebra:

- Singular value decomposition and pseudo inverse, QR factorization
- Algorithm and basic concepts of the Householder and Givens methods, application to overdetermined systems, least square problems

Course material

A.M. Quarteroni, R. Sacco, F. Saleri, Méthodes numériques, Algorithmes, analyse et applications, Springer, 2007.

Keywords

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	14,7 hrs	0,0 hrs	4 hrs

PROBABILITY

CORE COURSE

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Philippe CARMONA

Objectives

Understand the foundations of probability theory.
Learn techniques that are necessary for statistics.
Understand the different notions of convergence of a random variable.

Course contents

- Introduction to measure theory, classical theorems of integration with respect to a general measure.
- Real-valued random variables, usual probability laws, cumulative distribution function, expectation, variance, L^p spaces.
- Vector-valued random variables, independence, characteristic function.
- Convergence of random variables, almost sure, in probability, L^p, and in law. Illustration with the law of large numbers, the central limit theorem.

Course material

Barbe-Ledoux, Probabilités (EDP-sciences)

Keywords

Measure theory, Random variables, Convergence of random variables.

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	14,7 hrs	0,0 hrs	4 hrs

PROBABILISTIC NUMERICAL METHODS

CORE COURSE

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Bertrand MICHEL

Objectives

At the end of the course, the student will be able to understand and use stochastic methods for estimating quantities that are expressed as mathematical expectations. He/She will be able to propose and implement a simulation method for generating a sample from a certain distribution or a Markov chain allowing to infer the target quantity, and to evaluate the the precision of the estimation. Finally, he/she will be able to understand and use randomized numerical methods for solving high dimensional problems in scientific computing and data science which can not be treated with classical method in numerical linear algebra.

Course contents

The first part of this course deals with the main methods of simulation of random variables: generation of pseudo-random numbers, inverse cdf method, rejection method and simulation of Markov chain with finite state space. The course then presents the Monte Carlo and MCMC methods as well as variance reduction techniques.

The last part of the course deals with randomized numerical linear algebra methods for high dimensional problems. It will present the principles of parsimonious sampling and random projection methods, and their applications to performing algebraic operations, matrix factorization, the solution of least squares problems and data compression.

Course material

- Robert, C. and Casella, G. (2004). Monte Carlo Statistical Methods, second edition. Springer–Verlag, New York.
- Michael W. Mahoney, (2011). Randomized Algorithms for Matrices and Data, Foundations and Trends in Machine Learning, NOW Publishers, Volume 3, Issue 2, 2011

Keywords

Links with other programmes

Deterministic numerical methods

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	0,0 hrs	14,7 hrs	4 hrs

STOCHASTIC PROCESSES

CORE COURSE

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Philippe CARMONA

Objectives

The aim of this course is to describe the stochastic processes that are commonly used in the mathematical modeling of random phenomena which evolve over time (or space). We will study basic processes, show how they are involved in the modeling of systems, and finally solve some problems, mostly related to limit theorems.

Acquired skills: Establish a simple (or simplified) mathematical model of a complex random system. Illustrate the properties of this model by showing properties in finite horizon or asymptotic properties, and interpret these properties by simulations of random processes.

Course contents

1. Poisson and Renewal Processes
2. Markov Chains
3. Martingales and Brownian motion

Practical sessions: simulation of complex processes, illustration of limit theorems.

Course material

1. R. Durrett Essentials of stochastic Processes, Series: Springer Texts in Statistics, 2nd ed., 2012 ISBN-10: 1461436141
2. H. TIJMS. A First Course in Stochastic Models, Wiley, 2nd edition, 2003. ISBN-10: 047149880
3. E. Pardoux. Processus de markov et applications, Collection: Sciences Sup, Dunod 2007. EAN13 : 9782100512171

Keywords

Markov Chains, Poisson process, Martingale, Brownian motion

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	16,0 hrs	14,0 hrs	0,0 hrs	0 hrs

NUMERICAL ANALYSIS FOR PARTIAL DIFFERENTIAL EQUATIONS

NUMERICAL ANALYSIS & PROBABILITY TRACK

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Françoise FOUCHER

Objectives

At the conclusion of the course the student will:

- Recognise and distinguish the model equations (heat, transport, Poisson) and their PDE classification
- Compute finite difference schemes, and analyse their relevance according to the considered equation
- Prove the consistency, the stability and the convergence of a scheme.

Course contents

Introduction to PDEs:

- classification of linear second order PDE
- representation formula: separated variables for heat equation or waves equation on a bounded domain, characteristics method for the transport equation, Greens function for the Laplacian Finite difference method
- Poissons equation in 1D, properties of the Laplacian matrix in 1D (principle of the 2D case)
- Consistency and order, stability, convergence for the evolution PDE (transport and heat)
- Explicit and implicit schemes

Course material

- Grégoire Allaire. Analyse numérique et optimisation. Ellipses, 2005.
- Ionut DANAILA, Pascal JOLY, Sidi Mahmoud KABER, Marie POSTEL. Introduction au calcul scientifique par la pratique. Dunod, Sciences Sup, 2005.
- Daniel EUVRARD. Résolution numérique des équations aux dérivées partielles. Masson, 3rd edition, 1994.
- Mark H. HOLMES. Introduction to numerical methods in differential equations. Springer, 2007.
- Randall J. LEVEQUE. Finite difference methods for ordinary and partial differential equations. SIAM, 2007.
- Brigitte LUCQUIN. Equations aux dérivées partielles et leurs approximations. Ellipses, 2004.
- Bijan MOHAMMADI, Jacques-Hervé SAIAC. Pratique de la simulation numérique. Dunod, 2003.
- Lionel SAINSAULIEU. Calcul scientifique. Dunod, Sciences Sup, 2000.
- Eleuterio F. TORO. Riemann solvers and numerical methods for fluid dynamics. Springer, 3rd edition, 2010.

Keywords

Numerical analysis, PDE, numerical schemes, finite difference method

Links with other programmes

Partial Derived Equations, Variational Approximations of Partial Derived Equations

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	14,7 hrs	0,0 hrs	4 hrs

PARTIAL DIFFERENTIAL EQUATIONS

NUMERICAL ANALYSIS & PROBABILITY TRACK

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Mazen SAAD

Objectives

The aim of this course is to provide the main tools of mathematical PDE analysis arising from physics and mechanics models. For that, we provide fundamental classical theorems for a rigorous justification of variational approaches.

Course contents

1. Introduction: weak derivations, smooth functions
2. Sobolev spaces: injection, trace
3. Variational formulation for second-order elliptic equations: boundary condition (Dirichlet, Neumann, Fourier, etc)
4. Nonlinear first-order PDE: exact solutions.
5. Linear evolution equations: energy estimates, Galerkin method, maximum principle
6. Project

Course material

- JM GILSINGER, M.JAI. Eléments d'analyse fonctionnelle, Fondements et application de l'ingénieur. Presses Polytechniques et universitaires romandes
- H. BREZIS. Analyse fonctionnelle, Théorie et applications. Masson.
- B. Lucquin. EDP et leurs approximations. Mathématiqes à l'université, ellipses.
- L. C. Evans. Partial Differential Equations, Garduate Studies in Mathematics, Volume 19, AMS.

Keywords

weak derivatives, elliptic equations, boundary conditions, Linear evolution equations

Links with other programmes

Functional Analysis, Numerical analysis for partial differential equations, Variational approximations for partial differential equations

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	14,7 hrs	0,0 hrs	4 hrs

VARIATIONAL APPROXIMATIONS FOR PARTIAL DIFFERENTIAL EQUATIONS

NUMERICAL ANALYSIS & PROBABILITY TRACK

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Marie BILLAUD-FRIESS

Objectives

This course gives an introduction to the finite element method (FEM).
The main goal is to present the theory and the mathematical analysis of the FEM that is illustrated by the resolution of simple problems motivated by fluid mechanics.

Course contents

- Well posed variational problems (Lax-Milgram, Necas, Saddle point problems)
- Galerkin method
- Interpolation and finite element method
- Study of advection-diffusion equation
- Study of Stokes problem

Application: practical application and numerical simulation are undertaken in class with Freefem++ (http://www.freefem.org/ff++/)

Course material

A. Ern, JL Guermond, Theory and practice of finite elements, Springer

Keywords

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	12,0 hrs	14,0 hrs	4,0 hrs	0 hrs

DATA MINING

STATISTICS & DATA SCIENCE TRACK

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Lise Bellanger

Objectives

The volume of data generated and stored has increased dramatically in recent decades: the Big Data phenomenon. In this course we present methods to describe, explore and consolidate the information contained in large or high-dimensional data sets.

Data analysis includes a set of statistical methods to describe and consolidate the information contained in data sets. The application of these methods is often a necessary preliminary step before any statistical modeling.

Course contents

1. Introduction
2. Tools for the description of a sample. Representation of a sample by maps: factorial analysis
3. Principal component analysis (PCA)
4. Factorial correspondence analysis (AFC MCA)

Representation of a sample by classes: clustering techniques

5. Classification methods: Classification by partition, hierarchical clustering (AHC)

Practical sessions on the free software R

Course material

L. Bellanger, R. Tomassone, Exploration de données et méthodes statistiques: Data analysis & Data mining avec R. Collection Références Sciences, Editions Ellipses, Paris, 2014.
F. Husson, S. Lê, J. Pagès. Analyse de données avec R. Presses Universitaires de Rennes, 2009.
G. Saporta, Probabilités, Analyse des données. Editions Technip, Paris, 2011.

Keywords

Exploratory Data Analysis, Data Description, factorial methods, classification methods, Software R.

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	13,3 hrs	14,7 hrs	0,0 hrs	4 hrs

STATISTICAL INFERENCE AND LINEAR MODELS -

STATISTICS & DATA SCIENCE TRACK

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Bertrand MICHEL

Objectives

Construct statistical estimators for probabilistic models and establish asymptotic properties of the proposed methods.

Course contents

1. Statistical model
2. Functional estimation: empirical process, kernel density estimation
3. Point estimation: moment methods, maximum likelihood, delta-method, asymptotic properties
4. Confidence regions

Course material

[1] Held and Bové Applied Statistical Inference: Likelihood and Bayes. Springer Berlin Heidelberg (2014)

[2] Pawitan All likelihood statistical modelling and inference using Likelihood. Oxford University Press (2001)

Keywords

Estimation; Parametric model; likelihood

Links with other programmes

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	3	16,0 hrs	14,0 hrs	0,0 hrs	0 hrs

COMPUTATIONAL STATISTICS

STATISTICS & DATA SCIENCE TRACK

MATHEMATICS AND APPLICATIONS, ENGINEERING PROGRAMME SPECIALISATION

AUTUMN SEMESTER

Professor: Bertrand MICHEL

Objectives

This course gives an overview of the most important methods for Statistical Computing.

Course contents

1. Introduction to R and basic statistics
2. Linear models with R
3. Optimization for Statistics
4. Clustering and EM algorithms
5. Bootstrap

Course material

R for Statistics. Pierre-Andre Cornillon et.al. CRC Press.

An Introduction to Statistical Learning: with Applications in R. Gareth James, Daniela Witten, Trevor Hastie, Robert Tibshirani, Springer 2013

Keywords

R software, computational statistics , optimization, clustering, Bootstrap.

Links with other programmes

Statistical Inference and linear models - Statistical Learning - Advanced Statistical Learning

LANGUAGE	ECTS CREDITS	LECTURES	TUTORIALS	LABO	PROJECT
French	**3**	**16,0 hrs**	**14,0 hrs**	**0,0 hrs**	**0 hrs**